Cyclic shearing interferometer for collimating short coherence-length laser beams

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Until now there has not been an accurate method for measuring the radius of curvature, \( R \), of a short coherence-length light source, such as a short-pulse or broadband laser. We show that the easily aligned cyclic shearing interferometer (CSI) solves this problem. The CSI produces a stable fringe pattern from which \( R \) can be determined and can be used on beams with short coherence times down to 300 fs because the two beams in the interferometer follow nearly the same path. Comparison with data from a broadband XeCl laser (30-ps coherence time) confirms that the CSI performs as theory predicts.

**Key words:** Cyclic shearing interferometer, interferometer, laser collimation, equal path interferometer.

I. Introduction

In many experimental situations it is often necessary to collimate or, more specifically, measure the radius of curvature (\( R \)) of a light beam. To this end there are a number of methods, both geometric and interferometric, that have been developed and can be used to check the degree of collimation of a light beam. However, not all these methods are appropriate for collimating sources with short coherence lengths. A short synopsis of each method and its applicability to short coherence-length sources is given below.

To collimate a beam using the geometric approach, one simply needs to compare the size of the beam at two different points along the beam's axis. Although this method can be applied to sources with short coherence times, it is cumbersome because it requires a large distance to obtain moderate accuracy. Bass and Whittier have improved the geometrical method by using a technique with corner-cube backreflection. However, this improved technique still requires a large distance, \( \sim 15 \text{ m} \), to obtain collimation within the range \( 360 \text{ m} \leq R \leq \infty \).

A more compact method to check the degree of collimation of a light beam can be obtained by using the interferometric technique. The interferometers that have been used can be put into two categories; those that use beam splitters or shearing plates, and those that use diffraction gratings. The first category of interferometers includes a parallel shearing plate; a modified Michelson interferometer; a wedged, shearing-plate interferometer (often referred to as a collimation tester); a cyclic shearing interferometer and a modified wedged, shearing-plate interferometer. The second category, interferometers with diffraction gratings, consists of all interferometers of the Talbot type. They all use the moiré technique to produce intensity patterns in which collimation can be determined. Unfortunately, all interferometers except the modified Michelson and the CSI have inherent path-length differences between the two interfering beams, which deems them unusable for short coherence-length sources. Although the modified Michelson interferometer can be operated at a near-zero path-length difference it is susceptible to small mechanical vibrations and air currents. However, the CSI has inherent stability as well as a near-zero path-length difference making it ideal for collimating short coherence-length sources.

A survey of the early beginnings of cyclic interferometers is given by Hariharan. Four other papers have dealt with the CSI with little or no interest in the radius of curvature. The most recent of these papers discusses the stability of a CSI. The paper by Cordero-Davila et al. discusses a method to measure the radius of curvature of a converging wave front. Wenzel performed some initial calculations for a vertical shearing interferometer and discussed the possibility of using the interferometer for measuring the radius of curvature of a short coherence-length source.

This paper describes the CSI and its operation for
measuring the radius of curvature (or collimation) of a short coherence-length source. Before discussing the detailed analysis of CSI, a novel ray-tracing method is described and then applied to the wedged, shearing-plate interferometer (collimation tester) to re-derive the results of Riley and Gusinow. Rederiving these results is a good illustrative example of the ray-tracing method. Next, a general fringe pattern is derived for a shearing interferometer, and how the radius of curvature can be determined from measuring various parameters of the fringe pattern is shown. The CSI is then analyzed in detail. Finally, we discuss an experimental test of the CSI that uses a broadband XeCl laser.

II. Theory

The CSI, shown in Fig. 1, is a single-pass cyclic interferometer in which a single incident light beam is split into two beams that follow nearly the same path in the interferometer but in different directions. The two exiting beams interfere and produce a fringe pattern on a screen from which several parameters can be measured to determine the radius of curvature of the incident beam. The fact that the beams follow nearly the same path provides the CSI with two distinct advantages. The first advantage is that the CSI is much more stable to mechanical vibrations than other multi-optic interferometers, such as a Mach-Zehnder interferometer. The second advantage is that the two counterrotating beam path lengths will be nearly the same. A path-length difference very close to zero means that the CSI could be used with sources that have short coherence lengths (short coherence times), such as broadband lasers or short pulse-length lasers.

A. Ray-Tracing Method

A vector ray-tracing method was used to evaluate how the various optical surfaces of an interferometer can split an incident beam to produce the two beams that form a usable fringe pattern. The method, described by Herzberger, is easy to use and easy to implement on a computer. Herzberger showed that both Snell’s law and the law of reflection can be combined and written compactly in vector form given by

\[ S' = S + \gamma N. \] (1)

where the \( n \)'s are the indices of refraction for the two media.

Several parameters of an interferometer, such as the shear and the path-length difference between the exiting beams, can be found by using the following vector tracing formula:

\[ a' = a + D - \frac{S}{|S|}. \] (3)

In Eq. (3) \( a' \) and \( a \) are vectors from a fixed coordinate system that point to the ending and the starting points of a beam (\( a' \) and \( a \)), \( S \) is the beam direction vector, and \( D \) is the distance traveled between the two points.

The ending point \( a' \) cannot be found until the distance traveled, \( D \), is known. One can find \( D \) by observing that when the point \( a' \) lies on a given optic plane a line drawn from the point \( a' \) to a known point on the plane, \( P_p \), will be perpendicular to the plane’s normal. In mathematical terms this can be expressed as

\[ P_p a' \cdot N = 0. \] (4)

Therefore, to find the distance between the starting point of a beam and a plane defined by a normal and a point, one can put the components of \( a' \) from Eq. (3)
into Eq. (4) to get an expression for $D$ from known values.

After solving for $D$, one can put it into Eq. (3) to get $a'$, which points to the point in which the beam reflects or refracts off of the optic's surface, $a'$. To proceed, one takes the vector $a'$ to be the new vector $a$ in which the beam proceeds from along the new direction vector found from Eq. (1).

B. Review of Wedged, Shearing-Plate Interferometer

The wedged, shearing-plate interferometer\(^5\) can be used to measure the radius of curvature of a coherent light beam. This interferometer, also called a collimation tester, is often used to collimate a beam by checking that the radius of curvature of the beam is infinite. Although Riley and Gusinow\(^6\) have investigated the collimation tester and its fringe pattern, the details of their method of analysis of the two exiting beams was not given. Therefore, for completeness and as a simple illustrative example of the use of the direction vectors described in Subsection II.A, here we rederive the formulas for the shear and the tilt components between the two exiting beams of a collimation tester exactly. We see that the formulas that Riley and Gusinow derived rely on small-angle approximations but are appropriate for most experimental situations.

Figure 2 is a schematic of the collimation tester with its surface normals and the incident-beam direction vector. Note that the incident beam is in the $x$-$y$ plane. The tracing formula given by Eqs. (1) and (2) is used to trace the two beams through the interferometer. A top view of the beam direction vectors is shown in Fig. 3. Note that $S_1$ is simply reflected off the front surface and remains in the $x$-$y$ plane. However, the ray $S''$ will not lie in the $x$-$y$ plane because of the reflection off of surface 2, which has tilt about the $x$ axis. The results are given by

$$
S = -\sin \alpha x + \cos \alpha y,
S_1 = -\sin \alpha x - \cos \alpha y,
S' = -\sin \alpha x + (n^2 - \sin^2 \alpha)^{1/2}y,
S'' = -\sin \alpha x - (n^2 - \sin^2 \alpha)^{1/2} \cos(2\delta)y
- \left(n^2 - \sin^2 \alpha\right)^{1/2} \sin(2\delta)z,
S''' = -\sin \alpha x - (1 - n^2 + (n^2 - \sin^2 \alpha)\cos^2(2\delta))^{1/2}y
- \left(n^2 - \sin^2 \alpha\right)^{1/2} \sin(2\delta)z,
$$

where the index of refraction of air is 1 and the index of refraction of the optic material is denoted as $n$. The vertical tilt, $\theta$, between the exiting beams is found by comparing the $z$ components of $S''$ and $S'_1$. The relationship between the tilt and the radius of curvature is derived in Subsection II.C. Because $S_1$ has no $z$ component, we find that

$$
\sin \theta = (n^2 - \sin^2 \alpha)^{1/2} \sin 2\delta,
$$

or for small angles as Riley and Gusinow assumed,

$$
\theta = (n^2 - \sin^2 \alpha)^{1/2} 2\delta.
$$

The horizontal tilt, $\xi$, between the two exiting beams is found by subtracting the angle that $S$ makes with the $y$ axis from the angle that $S''$ makes with the $y$ axis. This angle is found to be

$$
\xi = \alpha - \arctan \left[ \frac{\sin \alpha}{(1 - n^2 + (n^2 - \sin^2 \alpha)\cos^2(2\delta))^{1/2}} \right].
$$

The above expression can be simplified by using a Taylor expansion of the cosine term and then an
expansion of the arctangent. These simplifications are possible because $\delta^2$ is typically small. After expanding and keeping the first interesting term, the simplified expression is found to be

$$\xi = 2\delta^2 \tan \alpha (\sin^2 \alpha - n^2).$$  \hspace{1cm} (9)

We see that $\xi$ can be ignored because it is a function of $\delta^2$, whereas $\alpha$ cannot be ignored because it is a function of $\delta$. Subsection II.C shows that $\xi$ must be small compared with $\alpha$ if the radius of curvature of the incident beam is to be determined accurately. The horizontal shear, $s$, which is needed to produce a usable fringe pattern is found by projecting the direction vectors onto the horizontal plane and finding $Q$ as shown in Fig. 3. After simple geometry, $Q$ is found to be

$$Q = \frac{t \sin \alpha \cos \delta + 1}{(n^2 - \sin^2 \alpha)^{1/2} \cos \delta},$$  \hspace{1cm} (10)

where $t$ is the average thickness of the interferometer. The horizontal shear is then given by the perpendicular distance between the beams,

$$s = Q \cos \alpha = \frac{t \sin(2\alpha)}{(n^2 - \sin^2 \alpha)^{1/2}} \frac{(\cos \delta + 1)}{2 \cos \delta},$$  \hspace{1cm} (11)

which can be reduced for small $\delta$ to give the solution of Riley and Gusinow,

$$s = \frac{t \sin 2\alpha}{(n^2 - \sin^2 \alpha)^{1/2}}.$$  \hspace{1cm} (12)

In most experimental situations the equation for the horizontal shear is never used because the shear is generally measured experimentally.

Vertical shear, $v_s$, is the displacement of the two exiting beams in the vertical plane. It is shown in Subsection II.C that the vertical shear is only significant when it is approximately the same size as the beam's diameter. In that case the exiting beams will simply not interfere because one beam is above the other. The vertical shear can be found by projecting the direction vectors onto the vertical plane. One finds

$$v_s = t(n^2 - \sin^2 \alpha)^{1/2} \sin \delta,$$  \hspace{1cm} (13)

where, again, $t$ is the thickness of the interferometer where the incident beam strikes.

For the two exiting beams to interfere, the coherence length of the incident source must be less than, or comparable with, the path-length difference introduced in the interferometer. A 1-cm-thick collimation tester with an incident beam striking at 45° will have a path-length difference, $\Delta L$, of 2.9 cm. This spatial difference corresponds to a temporal difference of $\Delta t = \Delta L/c = 96$ ps.

The results of Riley and Gusinow agree with approximations (7) and (12) when the small-angle approximations are used. In the sections below, these interferometer parameters are used to find a resulting fringe pattern, and finally, the radius of curvature of the incident light beam.

C. Derivation of the Fringe Pattern

Subsection II.B showed that a wedged, lateral shearing interferometer has two exiting beams with an arbitrary tilt and shear between them. For convenience the tilt and shear were broken into horizontal and vertical components: $\theta$, vertical tilt; $\xi$, horizontal tilt; $v_s$, vertical shear; and $s$, horizontal shear. The derivation of the interference fringe pattern is conducted in the same manner in which Riley and Gusinow derived their fringe pattern except that this paper includes two more parameters, $\xi$ and $v_s$ (described in Subsection II.B), which will be needed for the discussion of the CSI in Subsection II.D.

Figure 4 shows the two interfering beams exiting an interferometer; each is labeled with its own coordinate system. The two beams have scalar electric fields

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**Fig. 4.** The side and the top views of two rays exiting a generic interferometer with a coordinate system associated with each ray. $E_1$ and $E_2$ are the electric fields associated with the spherical wave fronts of the two rays. $\theta$ and $\xi$ are the vertical and the horizontal tilt components between the two rays, and $v_s$ and $s$ are the vertical and the horizontal shear components between the two rays. Int, interferometer.
for a uniform spherical wave front given by

\[ E_1 = E_0 \exp \left[ -\frac{ik}{2R(x^2 + y^2)} - ikz \right] \]

\[ E_2 = E_0 \exp \left[ -\frac{ik}{2R(x^2 + y^2)} - ikz' - i\beta \right] \]  

(14)

where \( R(z) \) is the radius of curvature, \( k \) is the propagation number \( 2\pi/\lambda \), and \( \beta \) is a constant phase shift. \( \beta = \pi \) for the collimation tester because one beam undergoes an internal reflection, whereas the other beam comes from an external reflection. It will be shown later that \( \beta = \pi \) for the CSI for a similar reason. \( R(z) \) can be treated as a constant in the typical case when the incident radius of curvature is much larger than the path dimensions in the interferometer. The transformation between coordinate systems is given by

\[
\begin{bmatrix}
  x' \\
  y' \\
  z' \\
\end{bmatrix}
=
\begin{bmatrix}
  \cos \xi & 0 & \sin \xi & 1 & 0 & 0 \\
  0 & 1 & 0 & 0 & \cos \theta & \sin \theta \\
  -\sin \xi & 0 & \cos \xi & 1 & 0 & \cos \theta \\
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
\end{bmatrix}
- (-D)
\]

(15)

where \( D \) is the extra distance traveled in one path of the interferometer. For small angles this becomes

\[
\begin{bmatrix}
  x' \\
  y' \\
  z' \\
\end{bmatrix}
=
\begin{bmatrix}
  1 & 0 & \xi & 1 & 0 & 0 \\
  0 & 1 & 0 & 0 & 1 & \theta \\
  -\xi & 0 & 1 & 0 & -\theta & 1 \\
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
\end{bmatrix}
- (-D)
\]

(16)

By keeping only linear, small-angle terms, we find that the derivation of the fringe pattern is easier and eliminates the worry of the transformation matrix order.

Because the total interfering electric field is given by

\[ E_r = E_1 + E_2 \]  

(17)

the field intensity will be

\[ I = E_r^*E_r \]  

(18)

An equation for the intensity pattern is found by changing the primed coordinates in \( E_2 \) to unprimed coordinates given by Eq. (16) then plugging \( E_1 \) and \( E_2 \) into Eq. (18). Neglecting the multiplicative constants, the intensity pattern can be simplified to

\[ I = \sin^2 \left[ \frac{k}{2R} \left( x(\xi - \xi z + \xi L) + \gamma (\xi z - \xi L + \xi R) \right) + \phi_0 \right] + \frac{\phi_0}{2}, \]  

(19)

where the interference intensity pattern was evaluated at a distance \( z = L \) from the interferometer. Again, for a large radius of curvature,

\[ R(L) = R(L + D) = R, \]  

(20)

where \( R \) is essentially a constant through the path of the interferometer. \( \phi_0 \) is a constant phase and has no effect on the fringe pattern other than shifting the whole pattern.

A typical fringe pattern is shown in Fig. 5. The vertical distance between fringes, \( d \), is found by finding points on the \( y \) axis that have a phase difference of \( \pi \). Using Eq. (19) we can show this to be

\[ d = \frac{\lambda}{\theta \left( 1 - \frac{L}{R} \right) + \frac{\phi_0}{R}} \]  

(21)

The slope of the fringes, \( \phi \), is the lines of the constant phase of the intensity formula Eq. (19), that is, \( y/x = \tan \phi = \text{constant} \). Using the definition for \( d \), we can find the slope of the fringes from Eq. (19) yielding

\[ \tan \phi = \frac{sd}{\lambda} + \frac{d\xi}{\lambda} \left[ 1 - \frac{L}{R} \right] - \phi_0. \]  

(22)

Note that the radius of curvature, \( R \), could not be easily found because of the existence of the horizontal tilt, \( \xi \). This small angle cannot be measured easily with an acceptable amount of accuracy but can be ignored when \( \xi \ll \text{s}/R \). For the collimation tester this is the usual case, so \( \xi \) is assumed to be zero in Eq. (22), and the radius of curvature can be solved for, thus yielding

\[ R = \frac{sd}{\lambda \tan \phi}. \]  

(23)

Typically, one measures \( \phi, s, \) and \( d \) experimentally so that the radius of curvature can be determined from Eq. (23) for a known central wavelength. We show below that \( \xi \) will not always be negligible for the CSI, so a scheme will be devised to eliminate \( \xi \) so that the CSI can be used with great accuracy.

![Fig. 5. Schematic of a typical fringe pattern showing the various important physical parameters. s and vs are the horizontal and the vertical shears, and d is the vertical distance between the fringes. φ is the horizontal tilt of the fringes.](image-url)
D. Cyclic Shearing Interferometer

The CSI is shown in full detail with the beam direction vectors in Fig. 6, with the exception of the infinitely thin beam splitter. The assumption of a thin beam splitter makes the calculation of the beam direction vectors much easier and leads to more readable equations. The effect of a real beam splitter is examined in Section IV. The method outlined in Subsection II.A was used to find the following beam direction vectors:

\[
\begin{align*}
S_1 &= \hat{x}, \\
S_2 &= \hat{y}, \\
S_3 &= 1 - 2 \cos^2 \gamma \cos \frac{\pi}{8} \hat{x} - \cos^2 \gamma \sin \frac{\pi}{4} \hat{y} - \sin 2\gamma \cos \frac{\pi}{8} \hat{z}, \\
S_4 &= \sin^2 \gamma \sin \frac{\pi}{4} \hat{x} + \left( \cos^2 \gamma - \sin^2 \gamma \sin \frac{\pi}{4} \right) \hat{y} - \sin 2\gamma \cos \frac{\pi}{8} \hat{z}, \\
S_5 &= S_{2c}, \\
S_6 &= \sin^2 \psi \hat{x} - \cos^2 \psi \hat{y} - \sin 2\psi \cos \frac{\pi}{4} \hat{z}, \\
S_{6c} &= \frac{\sqrt{2}}{2} \hat{x} + \frac{\sqrt{2}}{2} \cos 2\psi \hat{y} - \frac{\sqrt{2}}{2} \sin 2\psi \hat{z}, \\
S_{6c} &= -1 - i \left[ 1 + \frac{\sqrt{2}}{2} \gamma^2 + 2.61313 \gamma \psi + \psi^2 \right] \hat{x} + \left[ \frac{\sqrt{2}}{2} \gamma^2 + 1.08239 \psi - \psi^2 \right] \hat{y} + \left( -1 - \frac{\sqrt{2}}{2} \gamma^2 + 2.61313 \gamma \psi \right) \hat{y} + \left( -1 + \frac{\sqrt{2}}{2} \gamma^2 + 2.61313 \gamma \psi + \psi^2 \right) \hat{z}.
\end{align*}
\]

In Eqs. (24), \( S_{6c} \) refers to the \( N \)th counterclockwise direction vector, and \( S_{6c} \) refers to the \( N \)th clockwise direction vector. Some of the direction vectors were reduced to third order in small angles to have manageable equations in which the dependence on various angles is manifest.

The vertical tilt, \( \theta \), between the two exiting beams is found by subtracting the angles that the two exiting beams make with the \( x-y \) plane. These angles can be found by comparing the \( z \) components of the two exiting beams. Assuming small angles, the vertical tilt is found to be

\[
\theta = \sin \theta = 2 \sqrt{2} \psi + \left( -2 + \sqrt{2} \right) \gamma \psi + \gamma \psi \left( 2 \sqrt{2} - 4 \right) \cos \frac{\pi}{8} + \left( 4 \sqrt{2} - 4 \right) \sin \frac{\pi}{8}.
\]  

The horizontal tilt, \( \xi \), between the two exiting beams is found by comparing the difference between the angles the beams make with the \( y \) axis in the \( x-y \) (horizontal) plane. Note that both beams lie close to the \( y \) axis, which allows us to make small-angle approximations. The horizontal tilt is

\[
\xi = \frac{\gamma^2}{\sqrt{2}} \left[ 1 - \left( 1 + \frac{\sqrt{2}}{2} \gamma^2 \right) \right] - \left( -1 + \frac{\sqrt{2}}{2} \gamma^2 + \gamma \psi \left( 2 \cos \frac{\pi}{8} + 2 \sin \frac{\pi}{8} \right) \right) + \left[ 1 - \left( 1 + \frac{\sqrt{2}}{2} \gamma^2 \right) \right] + \left( -1 + \frac{\sqrt{2}}{2} \gamma^2 + \gamma \psi \left( 2 \cos \frac{\pi}{8} + 2 \sin \frac{\pi}{8} \right) \right) \left[ 1 - \gamma = 0 \right] \text{; the two roots are}
\]

\[
\psi = -\gamma \left[ \sin \frac{\pi}{8} + \frac{1}{2} \left( 2 + \sqrt{2} \right) \right] \approx -1.306 \gamma.
\]  

In Subsection II.C, specifically Eq. (22), it was shown that the horizontal tilt must be eliminated to use a shearing interferometer to measure the radius of curvature. \( \xi \) can be eliminated by using a relationship between \( \psi \), the tilt on the beam splitter, and \( \gamma \), the tilt on mirror 2, \( M_2 \). This relationship is established by setting \( \xi = 0 \); the two roots are

\[
\psi = -\gamma \left[ \sin \frac{\pi}{8} + \frac{1}{2} \left( 2 + \sqrt{2} \right) \right] \approx -1.306 \gamma.
\]
There will not be any horizontal tilt introduced as long as the interferometer is operated by using the relationship above. An experimental method for ensuring that inequality (28) is satisfied is presented in Section III. Because normal operation will require that the angles of the CSI follow the above relationship, it can be substituted into the inequality for the vertical tilt, relation (25), to give the simple relationship

$$s = 4 \sin \frac{\pi}{8} P',$$

where $P'$ is the translation of the mirror.

The third method involves rotating M1 and M2 about the vertical axis as shown in Fig. 9. The shear is found by tracing the beams around the CSI and

$$\theta = 2 \sqrt{2} \psi^3.$$  \hspace{1cm} (29)
finding their horizontal displacement on the x axis. The shear, s, was found to be

\[
s = (2 + \sqrt{2})L \left[ \sin \left( \frac{\pi}{4} + 2\beta \right) \left[ 1 + \tan \left( \frac{\pi}{8} + \beta \right) \right] - \cos \left( \frac{\pi}{4} + 2\beta \right) \left[ 1 + \cot \left( \frac{3\pi}{8} + \beta \right) \right] \right], \tag{32}
\]

where \( \beta \) is the rotation angle and \( L \) is the length of the short leg. This method requires care to make sure both mirrors are rotated the same amount, otherwise horizontal tilt would be introduced.

One important consideration is the path-length difference inherent in the CSI. The ability to measure the radius of curvature of a broadband beam requires that the path-length difference between the two exiting beams must be smaller than the coherence length of the light source. The path-length difference between the two exiting beams is found by using the vector-tracing method described in Subsection II.A. The calculation was performed assuming that the CSI was set up in the right-triangle configuration shown in Fig. 6. The origin \((0, 0, 0)\) was chosen to be where the incident beam strikes the beam splitter. \(M_1\) has one fixed point at \(P_{0_1} = (0, -L, 0)\), and \(M_2\) has a fixed point at \(P_{0_2} = (L, 0, 0)\). The beam direction vectors are given in Eqs. (24). By tracing a beam through the CSI, one can find the distance it travels to intercept the viewing plane. Once the path-length of each beam is known, subtracting the two will yield the path-length difference, \(\Delta L\). The alignment of the viewing screen for the purposes of this calculation is shown in Fig. 10. Note that the screen has its normal parallel to \(S_{sc}\), and a known point, \(P_{\text{screen}}\). This point, \(P_{\text{screen}}\), is where the counterclockwise beam strikes the beam splitter immediately before it exits the CSI. Because of the lengthy calculations, the research was conducted on a computer to give the numerical relationship

\[
\Delta L = 2.6678L \psi^2. \tag{33}
\]

\(\gamma\) was eliminated by using the relation given in Eq. (28), which is necessary to operate the CSI without horizontal tilt.

### III. Experimental

The CSI must be calibrated with a beam known to be collimated because the CSI has several parameters, such as tilts and positions of the optics, that must be set properly for its operation. The reference beam that was used was a He–Ne beam that was collimated after it was expanded from a spatial filter. The reference beam’s collimation was checked with a collimation tester. After the CSI was aligned with the reference beam, described in detail below, it operated the same with any other incident beam, with the exception of the fringe spacing, which is dependent on the wavelength [see Eq. (21)].

The alignment procedure is fulfilled in two segments. The first part is to align the CSI with all optics perpendicular to the table as shown in Fig. 1. When the optics are aligned so that the collimated, counter-rotating reference beams lie on top of each other, an interference pattern that does not have any fringes is produced. This pattern without fringes is called a null pattern, which implies that the two collimated beams exiting the CSI have no tilt between them. The operator must be sure that there is a null pattern and not just finely spaced fringes.

The second part of the alignment scheme is to adjust the interferometer so that the appropriate fringe pattern will be formed. Using a collimated He–Ne beam, we can set the tilt between the exiting beams, and, thus, the number of fringes visible, by simply varying \(\psi\). However, to avoid horizontal tilt, \(\gamma\) must be varied at the same time to satisfy Eq. (28). This is easy to do experimentally by varying \(\gamma\) to keep the fringes horizontal for the collimated reference beam. The sensitivity, the amount that the fringes tilt when the radius of curvature is changed, can be adjusted by changing the amount of shear by any of the techniques discussed in Subsection II.D [Eqs. (30)–(32)].

We used the setup shown in Fig. 11 to test the CSI. The distance \(L\) between the beam splitter and the two mirrors was 8 cm. The typical vertical distance \(d\) between the fringes produced with the collimated, He–Ne test beam is 1 cm. A simple calculation using Eqs. (21) and (29) and \(d\) given above tells us that \(\psi = 28\) mrad. Using Eq. (33) and \(\psi \) and \(L\) given above, we found that the pathlength difference between the two exiting beams was \(\Delta L = 170 \mu m\). This corresponds to a temporal difference of \(\Delta t = \Delta L/c = 564\) fs, which implies that a beam with a coherence time less than 564 fs will not produce a fringe pattern with good visibility. The reader should note that the path-length difference produced by the collimation tester (Subsection II.C) is much larger \((\Delta L \sim 2.9\) cm) than the path-length difference produced by the CSI in this.
It is shown in Section IV that a compact CSI has a path-length difference of only 90 \, \mu m.

The radius of curvature of the beam incident upon the CSI is changed by moving the collimating lens a distance \( \delta \) along the beam axis. The theoretical radius of curvature is found from the Gaussian lens formula,

\[
\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i},
\]

with \( s_o = f + \delta \), where \( \delta \) is the displacement of the collimating lens from collimation along the beam's axis and \( f \) is the focal length of the lens. The radius of curvature is simply the image distance, \( s_i \), minus the distance between the lens and the screen where the fringe pattern is produced. The latter distance is usually small compared with \( R \), therefore, it can be ignored in most experimental situations. \( R \) is found by using a binomial expansion and solving for \( s_i \). We find that

\[
R_{\text{theo}} = s_i = \frac{f^2}{\delta}.
\]  

The laser shown in Fig. 11 was a XeCl excimer that lases at 308 nm with a linewidth of 1 cm\(^{-1}\). A linewidth of one wave number corresponds to a coherence time of 15-ps (FWHM). The spatial filter produces a Gaussian beam that is then passed through a collimating lens with a focal length measured to be 2028 ± 4 mm by using a method described by Malacara. After the beam passed through the CSI, it produced a visible pattern on a thin-sheet UV converter. The visible fringe pattern was then stored in computer memory for analysis by using a video camera and a frame grabber. The stored frames were analyzed with the help of the computer and a least-squares program to find the parameters \( s, d, \) and \( \tan(\phi) \), which are used to find \( R_{\text{exp}} \) from the individual fringe patterns by using Eq. (23). The visibility of the fringes was nearly 1 signifying that the coherence length of the laser was much longer than the temporal difference between the two exiting beams \( \Delta t = 565 \, \text{fs} \). In contrast, no fringes were produced by a collimation tester because its path-length difference (\( \Delta t \geq 96 \, \text{ps} \)) is much larger than the coherence time of the laser.

The theoretically computed \( R_{\text{theo}} \) [approximation (34)] is plotted with the experimentally determined \( R_{\text{exp}} \) [Eq. (23)] as a function of \( \delta \) in Fig. 12. The error bars for the experimental points come from the uncertainty in measuring \( s, d, \) and \( \tan(\phi) \). The agreement between theory and experiment is excellent.

IV. Discussion

The theoretical analysis of the CSI was conducted assuming that the beam splitter was infinitely thin. In practice this is not true, as real beam splitters have both thickness and a wedge angle between the two surfaces. The nonzero thickness of the beam splitter in itself does not introduce any new tilts on the interfering beams. However, the effect of the nonzero thickness coupled with a wedge produces some tilts. A wedge that is oriented so that it deflects a beam vertically will simply affect the vertical tilt between the two exiting beams and hence the vertical shear. A wedge that is oriented into the horizontal plane will produce a horizontal tilt that will have to be counteracted by adjustments to \( \gamma \) or \( \psi \). The effect of a wedge oriented into the horizontal plane is small for typical amounts of wedge because the horizontal tilt is proportional to the square of the wedge angle. It should be noted that the effects of the wedge are small in any orientation and a small amount of adjustment on \( \gamma \) and \( \psi \) can counteract these effects.

Another effect of a real beam splitter is a small secondary reflection off its back surface that will produce lighter, secondary fringes that may confuse the operator when setting up the CSI or hamper the operator when reading the fringe pattern. The secondary fringes can be distinguished from the desired fringes because they will rotate if the beam splitter is rotated. The secondary fringes can be eliminated by...
having either a good antireflection coating on the back surface or by having a beam splitter with a wedge less than 1/2 fringe.

We found that one can collimate a laser beam that has a diameter down to 1.5 cm without taking special care. However, when collimating a beam with a diameter between 1.5 and 0.5 cm, one must reduce the amount of shear, which is usually approximately 1 cm, so that the beams will overlap to form a fringe pattern. For small diameter beams, one may also want to increase the tilt between the exiting beams to reduce the spacing between the fringes. However, the effect of reducing the shear and the spacing between the fringes will decrease the sensitivity of the interferometer. The relation between the slope of the fringes and the radius of curvature can be seen in Eq. (23). When $s$ and $d$ are made smaller for a given $R$, the slope of the fringes, $\tan \phi$, will be smaller. This reduced sensitivity will increase the uncertainty in $R$.

Vertical shear may be of concern if it is approximately the same size as the incident beam’s diameter. This might happen if the cyclic path-length in the CSI is large, or if beams of small diameters are used. There are three ways to reduce or eliminate the vertical shear. The first method is to reduce the size of the legs of the CSI, which will reduce the beams’ path-lengths, in turn reducing the vertical shear. This will reduce, but not eliminate, the vs. The next method is to use a beam splitter with a wedge oriented so that it deflects a beam vertically, which would counteract the vs produced by the normal operation of the CSI. The final method that can be used is to place a window in the long leg of the CSI as shown in Fig. 13. Vertical shear can be varied if the window is rotated about an axis that is in the horizontal plane perpendicular to the beams axes. All the methods described above can be used separately or together to reduce or eliminate the vertical shear.

A compact one-piece design of the CSI would provide the user with two distinct advantages over a CSI constructed out of several pieces of optics. One such design is shown in Fig. 14. The first advantage is the smaller size of the compact unit. A smaller CSI implies a smaller path-length difference between the two exiting beams [Eq. (33)], which means that the smaller CSI would produce usable fringe patterns with beams that have a shorter coherence length. A compact unit that is built to accommodate a 0.75-cm beam will have a path-length difference of $\Delta L \approx 90 \mu m$ ($\Delta t \approx 300$ fs). This path-length difference was computed by multiplying Eq. (33) by the index of refraction to find the optical path-length difference. $\psi$, used in Eq. (33), was chosen so that two fringes would be produced in the fringe pattern. The distance between the beam splitter and the mirrors, $L$, was chosen to be 1.5 cm in order to use the entire width of the beam for the fringe pattern. This path-length difference implies that a beam with a coherence time greater than 300 fs will produce a usable fringe pattern with two fringes.

The second advantage is one of convenience. A one-piece CSI would permit the user to simply place the CSI into a beam line to measure the radius of curvature. The sensitivity of the CSI could be adjusted by simply rotating it. The sensitivity changes with a rotation about a vertical axis with changes of shear.

V. Conclusion

A survey of interferometers showed that, up until now, there was not an accurate method of which we are aware that could be used to collimate or, more specifically, measure the radius of curvature of a short-pulsed or broadband laser. After reviewing the collimation tester with a novel ray-tracing scheme, the method was applied to the CSI. We found that the CSI could be operated to produce a fringe pattern much like that of the collimation tester. The major difference between the two interferometers is that the CSI has a small path-length difference between the two exiting beams. The collimation tester, on the other hand, has a relatively large path-length difference between its exiting beams. We found that the CSI can be operated with a path-length difference down to $\Delta L \approx 90 \mu m$ ($\Delta t \approx 300$ fs), but the collimation tester has a path-length difference of 2.9 cm. The small path-length difference implies that the radius of curvature, $R$, could be measured from a beam that

Fig. 13. Use of a window in the long leg of the CSI to eliminate vs.

Fig. 14. Compact, one-piece design of the CSI. The tilts of the reflecting surfaces must be ground to resemble a CSI made from separate pieces. A.R., antireflection.
has a coherence length of 90 μm (Δt = 300 fs) or greater. Experimental results showed that the CSI behaved as predicted; the comparison between the theoretical and the experimental results was excellent. We also showed that the CSI could be built as a compact, one-piece unit for the shortest path-length possible and for convenience.

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References